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# CALCULATION OF CONDUCTOR TENSIONS OF A TENSION ANGLE SUPPORT TAKING INTO ACCOUNT SHIFTING OF THE POLE BODY

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## 1. SUMMARY

*The problem considered here was treated in Ref. [1] for particular case of tension-line support. This paper deals with the influence of shifting (declination) of a tension-angle support on conductor tensions and sags in neighboring sections of an overhead distribution line. The overhead line designed with reinforced concrete pole and steel reinforced aluminum conductors is analyzed. Shifting of tension–angle support is caused either by changing of conductor temperature and/or different maximum working conductor tensions in two neighboring sections defined by designer of overhead line. Shifting of the support caused only by different conductor tensions at critical temperature is treated here. Influence of temperature on conductor tensions and sags will be the subject of some other paper. It is pointed out that problem which appears in Ref. [1], can be treated as a special case of consideration explained here. We hope that the matter is comprehensively treated.*

## 2. INTRODUCTION

In overhead line design, a maximum working values of conductor tensions in sections are defined on critical temperature (-20 $^{0}$C or –5 $^{0}$C with ice loading). On the basis of previously defined values, sags in all spans of a section, ground clearance, clearance to obstacles and some other values are calculated. It is usualy assumed that the support is ideally immobile, i. e. that pole reinforced body is ideally stiff. The problem is said to be idealized and simplified.

Does the actual distribution of horizontal projections of conductor tensions in neghboring sections of tension angle / flying angle support differ from that of assumed by designer, if pole body is not treated as stiff, but elastic (flexible)?

It is evident that the support is moved into the direction of resultant vector to the point where the equilibrium is achieved. It is a new steady state which takes into consideration the vector of pole reaction. In that state, according to [2], the sum of horizontal projections of vectors applied to support must be equal to zero. This paper is trying to give an answer to these questions.

On the basis of type testing in »Institute for Materials Exploring« it is pointed out in Reference [1] that a pole body can not be treated as absolutely stiff, but a coefficient of pole stiffness should be defined, if a greater accuracy were desirable.

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Results of calculation, which are taking into account foregoing condition given in preceding paragraph, will be presented later, (in section 5), using convenient numerical example of an overhead line with reinforced concrete poles. Because of that, abstract of results of the type testing of reinforced pole 12m/1000daN with round transversal section is shown in Table 1. As it can be seen, the top humper shifting is not negligible.

**TABLE 1 – ABSTRACT FROM TYPE TESTING RESULTS OF A 12m/1000daN POLE BODY GIVEN IN THE MANUFACTURER'S SPECIFICATION SHEETS**

| FORSE [daN] | TOP HUMPER SHIFTING [mm] | ACTUAL TOP HUMPER SHIFTING [mm] | REMAINING TOP HUMPER SHIFTING [mm] |
|---|---|---|---|
| 0 | 61 | 35 | 26.2 |
| 500 | 247 | 176 | 71.0 |
| 1000 | 442 | 348 | 93.7 |
| 1300 | 578 | 462 | 11.9 |
| 1600 | 728 | 616 | 111.6 |
| 1800 | 830 | 677 | 153.0 |

Influence of support shifting can be mathematically evaluated by a coefficient of stiffness defined as:

$$C = \frac{F_{rat}}{\Delta x_{rat}}$$

where $F_{rat}$ is the rating value of the forse in $daN$, and $\Delta x_{rat}$ is the rating pole shifting in $m$. It should be emphasized that $F_{rat}$ is the force which, in accordance to Ref. [2], appears on the top of a pole. Stiffness of a pole body can be defined as ratio of difference of forse to infinitesimal shifting as follows:

$$C = \left.\frac{\Delta F}{\Delta x}\right|_{F=0,\, x=0} = \left.\frac{dF}{dx}\right|_{(0,0)} \qquad (1)$$

In numerical example in Sec. 5 the definition of $C$ given by expression (1) is used.

## 3. ASSUMPTIONS AND SIMPLIFICATIONS

A tension angle support with preceding and succeeding span are considered. The following assumptions and simplifications are introduced to emphasize the essence of problem.

- spans are horizontal,
- a line angle is given,
- an overhaed line with steel reinforced aluminum conductor is considered,
- supports of catenary conductor on preceding and succeeding pole are immobile. It means that changing in lengths of neighboring spans is exclusively influenced by support shifting of pole considered.

## 4. DERIVATION OF MATHEMATICAL MODEL

A pole placed in the centre of the reference system with two perpendicular axes, $X$ and $Y$, is sketched in Fig. 1. Lengths of neighboring spans of sections are denoted as $a_1$ and $a_2$, conductor tension in those spans as $\sigma_1$ [1] and $\sigma_2$, respectively, and the line angle as $\alpha$. A shifted point of support has coordinate $x$ and $y$. The shifting of support causes $a_1$, $a_2$, $\sigma_1$, $\sigma_2$ and $\alpha$ to change their values into $\tilde{a}_1$, $\tilde{a}_2$, $\tilde{\sigma}_1$, $\tilde{\sigma}_2$ and $\tilde{\alpha}$, respectively. As in Ref. [2], all forces that act on the pole body which is considered, are reduced on equivalent values that act on the top of the pole.

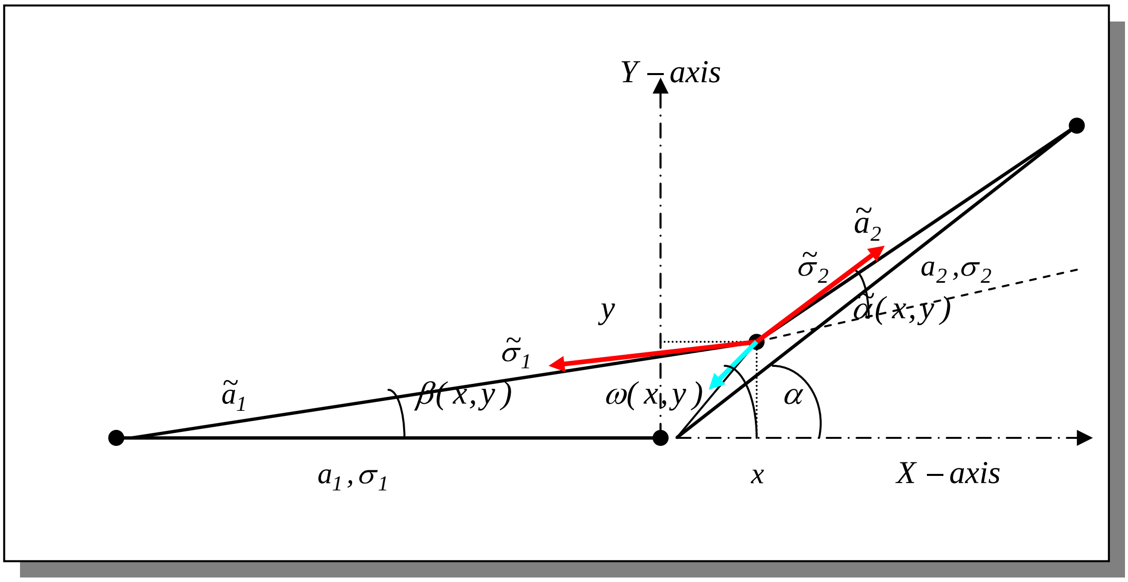


**FIGURE 1 – A TENSION ANGLE POLE WITH TWO NEIGHBORING SPANS**

Adding of force projections on $X$ -axis and $Y$ -axis that act on a shifted support, gives two equations:

$$F1(x,y,\tilde{\sigma}_1,\tilde{\sigma}_2) = k\,\tilde{\sigma}_2 S\cos(\tilde{\alpha}_{(x,y)} + \beta_{(x,y)}) - C\sqrt{x^2+y^2}\cos\omega_{(x,y)} - k\,\tilde{\sigma}_1 S\cos\beta_{(x,y)} = 0 \qquad (2)$$

$$F2(x,y,\tilde{\sigma}_1,\tilde{\sigma}_2) = k\,\tilde{\sigma}_2 S\sin(\tilde{\alpha}_{(x,y)} + \beta_{(x,y)}) - C\sqrt{x^2+y^2}\sin\omega_{(x,y)} - k\,\tilde{\sigma}_1 S\sin\beta_{(x,y)} = 0 \qquad (3)$$

In preceding equations $S$ is the cross sectional area of catenary conductor and $k$ is the coefficient of reduction, which has been determined by the relation, (see Ref. [2]):

$$k = 3 - 2\cdot\frac{h}{L_{pole}}$$

where is:
$h$ - difference of heights (altitudes) from upper to lower points of support of triangular conductor configuration. ($h=0$, in case of single circuit horizontal configuration),
$L_{pole}$ - pole body height.

Values $\tilde{\alpha}_{(x,y)}$, $\beta_{(x,y)}$ and $\omega_{(x,y)}$ are determined in Ref. [3][2], and are given with following expressions:

$$\tilde{\alpha}_{(x,y)} = arctg\left[\frac{(a_2\sin\alpha - y)(a_1+x) - y(a_2\cos\alpha - x)}{(a_2\cos\alpha - x)(a_1+x) + y(a_2\sin\alpha - y)}\right]$$

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[1] In the hole text letter $\sigma$ always means the horizontal component (projection) of conductor tension, that is $\sigma = \sigma_{horizontal}$.

[2] In Ref. [3], values $\alpha(x,y)$, $\beta(x,y)$ and $\omega(x,y)$ are calculated using complex algebra for ganeral case of overhead line with $N$ poles, which are either tension-angle or tension-line type.

$$\beta_{(x,y)} = arctg\left(\frac{y}{a_1 + x}\right)$$

$$\omega_{(x,y)} = arctg\left(\frac{y}{x}\right)$$

Using the expression which relates the difference of catenary length to corresponding difference of conductor tension in spans $a_1$ and $a_2$, yields:

$$\tilde{L}_1 - L_1 = L_1 \cdot \frac{1}{E} \cdot (\tilde{\sigma}_1 - \sigma_1) \quad (4)$$

$$\tilde{L}_2 - L_2 = L_2 \cdot \frac{1}{E} \cdot (\tilde{\sigma}_2 - \sigma_2) \quad (5)$$

Determining the catenary's lengths $\tilde{L}_1$, $\tilde{L}_2$, $L_1$ and $L_2$ in terms of their corresponding span lengths $\tilde{a}_1$, $\tilde{a}_2$, $a_1$ and $a_2$ and substituting into above equations transforms the equations (4) and (5) and gives

$$F3(x,y,\tilde{\sigma}_1,\tilde{\sigma}_2) = \frac{2\tilde{\sigma}_1}{p} sh(\frac{\tilde{a}_1(x,y)p}{2\tilde{\sigma}_1}) - \frac{2\sigma_1}{p} sh(\frac{a_1 p}{2\sigma_1}) = \frac{2\sigma_1}{p} sh(\frac{a_1 p}{2\sigma_1}) \frac{1}{E}(\tilde{\sigma}_1 - \sigma_1) = 0 \quad (6)$$

$$F4(x,y,\tilde{\sigma}_1,\tilde{\sigma}_2) = \frac{2\tilde{\sigma}_2}{p} sh(\frac{\tilde{a}_2(x,y)p}{2\tilde{\sigma}_2}) - \frac{2\sigma_2}{p} sh(\frac{a_2 p}{2\sigma_2}) = \frac{2\sigma_2}{p} sh(\frac{a_2 p}{2\sigma_2}) \frac{1}{E}(\tilde{\sigma}_2 - \sigma_2) = 0 \quad (7)$$

In equations (6) and (7) values $\tilde{a}_1(x,y)$ and $\tilde{a}_2(x,y)$ are the span lengths after shifting the point of support from imaginary position $(0,0)$ to position $(x,y)$, and is determined in Ref. [3] by following expressions:

$$\tilde{a}_1(x,y) = \sqrt{(a_1 + x)^2 + y^2} \quad (8)$$

$$\tilde{a}_2(x,y) = \sqrt{(a_2 \cos\alpha - x)^2 + (a_2 \sin\alpha - y)^2} \quad (9)$$

Now, equations (2), (3), (6) , and (7) can be considered as a set of equations:

$$\begin{cases} F1(x,y,\tilde{\sigma}_1,\tilde{\sigma}_2) = 0 \\ F2(x,y,\tilde{\sigma}_1,\tilde{\sigma}_2) = 0 \\ F3(x,y,\tilde{\sigma}_1,\tilde{\sigma}_2) = 0 \\ F4(x,y,\tilde{\sigma}_1,\tilde{\sigma}_2) = 0 \end{cases} \quad (10)$$

An array $(x,y,\tilde{\sigma}_1,\tilde{\sigma}_2)$ is the solution which simultaneously satisfies the foregoing system of nonlinear equations. On the basis of physical nature of problem, it is evident that set of equations has only one array $(x,y,\tilde{\sigma}_1,\tilde{\sigma}_2)$ which simultaneously satisfies all equations (10). When values $x$ and $y$ are calculated, the pole shifting $\Delta z = |\Delta\vec{z}|$, (also reduced on the top of the pole body) and shifting – angle $\omega$ [3] can be found as:

$$\Delta z = \sqrt{x^2 + y^2}, \quad (11)$$

[3] A shifting-angle $\omega$ is defined as angle between vector $\Delta\vec{z}$ and $X$ - axis.

$$\omega = tg^{-1}\left(\frac{y}{x}\right) \tag{12}$$

Substituting the expressions (8) and (9) into the wellknown relation for sags, those can also be evaluated.

System of equations (10) cannot be analytically solved. Hence, some of iterativ numerical method of solving must be used in order to give approximate values for $(x, y, \tilde{\sigma}_1, \tilde{\sigma}_2)$. In numerical example of calculation, the MATHCAD software is used to solve the set of equations. Drawing the results of computation has also been performed by the same software.

In order to solve the system of equations (10) the initial value of anknown array $(x, y, \tilde{\sigma}_1, \tilde{\sigma}_2)$ must be defined. The convergence of iterativ proccess depends on those initial values, which must be taken into consideration for solving the system.

## 5. EXAMPLE OF COMPUTATION

An overhead line designed with reinforced pole body and steel reinforced aluminum conductor in triangular configuration is considered in numerical example. The length of pole body is $L_{pole} = 12\,m$, and the type of conductor used is $50/8\ mm^2$. The results of calculation of conductor tensions in two neighboring sections and parameters of pole shifting is shown in Table 2. The sag can be treated as equivalent sag of a section.

**TABLE 2 – THE RESULTS OF CALCULATION OF CONDUCTOR TENSIONS AND PARAMETERS OF POLE SHIFTING FOR TENSION – ANGLE SUPPORT**

| INPUT DATA | |
|---|---|
| Span length $a_1$ in $m$ | 80 |
| Span length $a_2$ in $m$ | 80 |
| Specific conductor weight $p$ in $daN/m \cdot mm^2$ | 0.00348 |
| Pole body length $L_{st}$ in $m$ | 12 |
| The difference in height from upper to lower support $h$ in $m$ | 1.4 |
| Coefficient of conductor elacticity $E$ in $daN/mm^2$ | 8100 |
| ACSR conductor 50/8, equiv. cross section area $S$ in $mm^2$ | 56.3 |
| Coefficient of pole body stiffness $C$ in $daN/m$ | 8000 |
| Conductor tension in span 1 $\sigma_{1,max,work}$ in $daN/mm^2$ | 6 |
| Conductor tension in span 2 $\sigma_{2,max,work}$ in $daN/mm^2$ | 9 |

It is to be note that coefficient of pole body stiffness $C$ in Table 2 is expressed in $daN/m$. This unit is in accordance with the other units using in (2), (3), (6) and (7). Nevertheless, the support shifting $\Delta z$ depicted in Fig. 3 is expressed in millimeters for reason of convenience.

**OUTPUT DATA**

| angle $\alpha$ [°] | $x$ [mm] | $y$ [mm] | $\Delta z$ [mm] | $\omega$ [°] | $\tilde{\sigma}_1$ [daN/mm$^2$] | $\tilde{\sigma}_2$ [daN/mm$^2$] |
|---|---|---|---|---|---|---|
| 0 | 13.00 | 0.000 | 13.000 | 0.000 | 7.106 | 7.793 |
| 2 | 13.00 | 5.124 | 13.973 | 21.512 | 7.099 | 7.785 |
| 4 | 13.00 | 10.00 | 16.401 | 37.569 | 7.076 | 7.762 |
| 6 | 12.00 | 15.00 | 19.209 | 51.340 | 7.038 | 7.725 |
| 8 | 12.00 | 20.00 | 23.324 | 59.036 | 6.985 | 7.673 |
| 10 | 11.00 | 25.00 | 27.313 | 66.251 | 6.919 | 7.608 |
| 12 | 9.954 | 30.00 | 31.608 | 71.644 | 6.841 | 7.531 |
| 14 | 8.914 | 34.00 | 35.149 | 75.309 | 6.751 | 7.442 |
| 16 | 7.746 | 38.00 | 38.781 | 78.479 | 6.650 | 7.343 |
| 18 | 6.464 | 42.00 | 42.495 | 81.251 | 6.541 | 7.235 |
| 20 | 5.079 | 46.00 | 46.280 | 83.699 | 6.423 | 7.120 |
| 22 | 3.604 | 49.00 | 49.133 | 85.788 | 6.299 | 6.998 |
| 24 | 2.051 | 53.00 | 53.040 | 87.784 | 6.170 | 6.871 |
| 26 | 4.316 | 56.00 | 56.166 | 85.593 | 6.037 | 6.740 |

Results of calculating from Tab. 2 are shown in Fig. 2 and Fig. 3. Conductor tensions $\tilde{\sigma}_1$ and $\tilde{\sigma}_2$ in $daN/mm^2$ as a function of line angle $\alpha$ in degree are shown in Fig 1. It is to be seen from the curves, that conductor tension in span 2 which designer of an overhead line specifies at a value of $\sigma_2 = 9\,daN/mm^2$ (considering the pole body to be immobile) is decreased. The »new« velues of conductor tension in span 2, $\tilde{\sigma}_2$, for different values of the line angle $\alpha$, is calculated. Thus, we see that conductor tension $\tilde{\sigma}_2$ depends on line angle $\alpha$. Opposite of that the conductor tension in span 1 at immobile pole body, $\sigma_1 = 6\,daN/mm^2$, increases. It is evident from Fig. 2. that decreasing of conductor tension $\tilde{\sigma}_1$ in span 1 causes its equality to $\sigma_1 = 6\,daN/mm^2$, i. e. $\tilde{\sigma}_1$ is approhimately equal to $\sigma_1$. For given input data of calculation this occurs at higher values of line angle, ($\alpha \geq 25$ degrees).

This can be explained as follows: From geometrical reasons it is clear that projection on $X$ - axis of support shifting $\Delta\vec{z}$ becomes equal to zero. This practically means that there is no changing in span length $a_1$. Thus $\tilde{\sigma}_1 \approx \sigma_1$ is consequence of $\tilde{a}_1 \approx a_1$.

Shifting of point of support $|\Delta\vec{z}|$ in $mm$ and angle of shifting $\omega$ in degrees versus line angle $\alpha$ in degrees is shown in Fig. 3. Declination $|\Delta\vec{z}|$ increases with increasing the line angle $\alpha$ because of the increasing of resultant force of conductor tensions which operate on pole body.

We are interested in comparison the results of computing given in this paper with the results obtained in Ref [1], where the same problem was being solved for the case of line tension pole, using different method and approximating the hyperbolic functions with the corresponding polynoms of third order. Comparison of results is shown in Table 3.

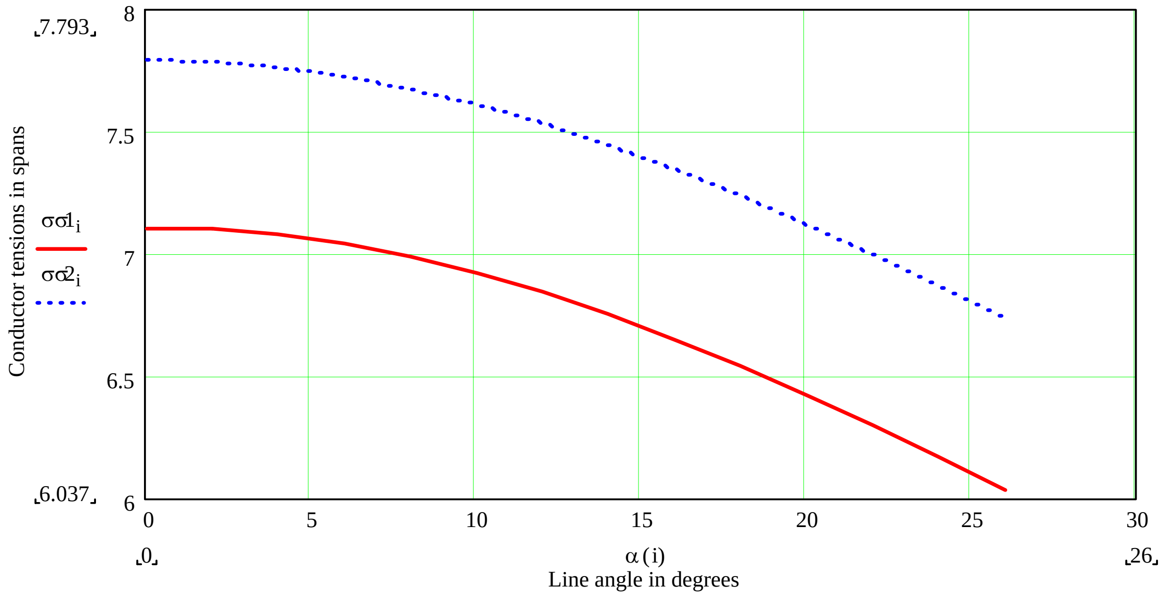


**FIG. 2. CONDUCTOR TENSIONS $\tilde{\sigma}_1$ AND $\tilde{\sigma}_2$ IN $daN/mm^2$ VERSUS LINE ANGLE $\alpha$ IN DEGREES**[4]

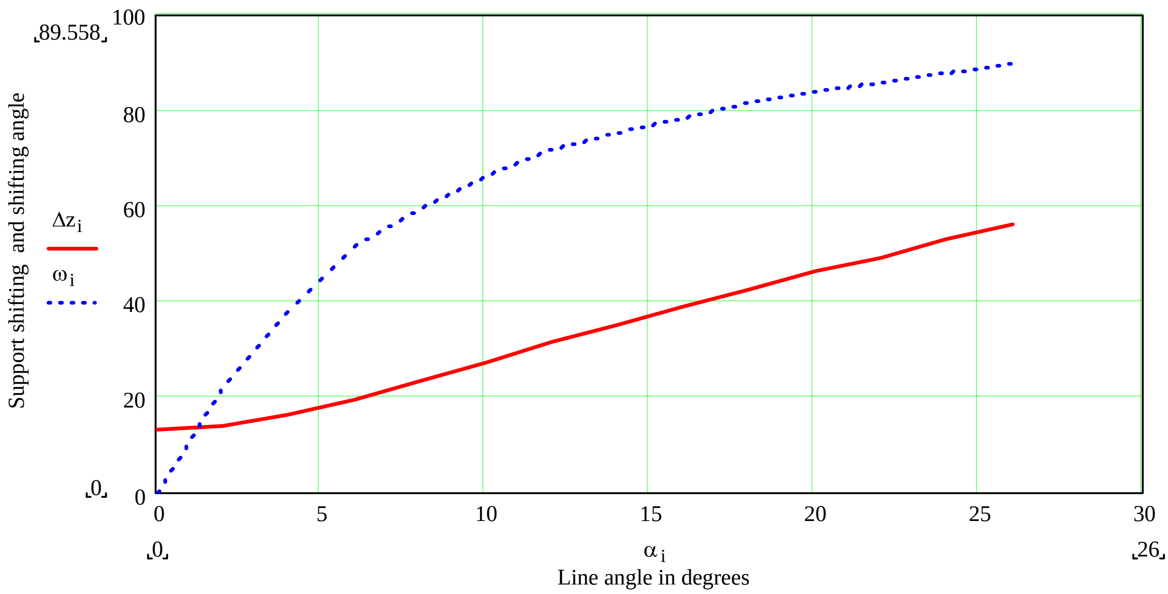


**FIG. 3 – SUPPORT SHIFTING $|\Delta\vec{z}|$ IN $mm$, AND SHIFTING ANGLE $\omega$ IN DEGREES VERSUS LINE ANGLE $\alpha$ IN DEGREES.**

**TABLE 3 - COMPARISON OF RESULTS OF COMPUTING BY THE METHOD GIVEN IN REF. [1] AND BY FOREGOING METHOD.**

| Input data: $\sigma_1 = 7\,daN/mm^2, \sigma_2 = 9\,daN/mm^2, \alpha = 0^0$ | | | |
|---|---|---|---|
| Results of calculating | $\tilde{\sigma}_1$ $daN/mm^2$ | $\tilde{\sigma}_2$ $daN/mm^2$ | $\Delta z$ $mm$ |
| by the method given in Ref. [1] | 7.757 | 8.204 | 8.708 |
| by the foregoing method | 7.759 | 8.207 | 8.477 |

[4] Curves are plotted using MATHCAD and symbol $\sigma\sigma$ means the same as $\tilde{\sigma}$ .

## 6. REFERENCES

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