TRANSFIGURATION METHOD OF RADIAL ELECTRICAL NETWORKS

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1. Introduction

Within electrical distribution network analysis, their complex configuration, of meshed type or tight-meshed type, even when keeping an arborescent power flow, lead to a higher difficulty in the power flow study or in applying certain optimization methods of operating state, methods based on mathematical patterns or on the artificial intelligence techniques [1]. In order to overcome this difficulty, in technical literature there are numerous attempts of simplifying things, all mainly based on the reduction of the complex real networks to an equivalent one, having a simple structure, on which to apply analyses or methods of operating state optimization easier. A good example of this may be considered the method proposed in [2] for optimal voltage control within a distribution network of complex configuration. According to this method, the optimal value of the voltage on the nodes of the descending transformer substation that supplies the network is established in accordance with the optimal value of the voltage in a node of the network called characteristic or representative node, whose equivalent electrical distance to the nodes of the supply substation is called image impedance. This is actually the equivalent impedance of a fictitious line that links the characteristic node to the source, line that is traversed by the entire load of the network. The real and reactive components of this impedance result from the minimization condition of an indicator of voltage quality, written analytically for the characteristic node. Paper [3] can also be mentioned here, where there are presented two methods of reduction of a feeder of complex configuration, which feeds several loads, to a single consumer node where the entire load of the network is connected, node that is linked to the source through a fictitious equivalent electrical line. The length of this line is calculated differently, if the equalization from the point of view of the maximum voltage drop is needed, of total real power losses on the considered feeder, respectively. A hybrid method of equalization, valid at the same time for both criteria is presented in [4] with the mention that the load oh the network is distributed in two nodes fed through the same fictitious feeder, one node being placed at the end of the network. In [5] a method similar to the previous one is presented, method obtained through the improvement of this one, in the way that the second node on the fictitious feeder has the role of allowing a correction to be made which performs the equivalence valid for both criteria, this node being a consumer or a source, according to the situation. The reduction to a single fictitious feeder, this time in order to obtain an equivalence which helps evaluate the stability of the voltage or to detect possible voltage collapses in the real network is presented in [6].

2. Raising the problem

When dealing with problems of optimizing the operating states of electrical networks, the values of consumer nodes voltages or the deviation of these values from the reference values, or the value of the real power losses, constitute elements on the basis of which the objective function or the restriction are made. It can be said that these elements characterize the state of the network and define a link relation between the source and the nodes of the network. However, this link is not a direct one because between the source and any node of the network there are usually more network elements (lines, transformers) having very different characteristics. The concept presented in this paper refers to a method of transfiguration of an arborescent network into an equivalent network, where the relation between each node and the source becomes a direct one, through a single fictitious one. In other words, to each consumer node, or in general, to each interest node is associated a direct link with the source, an individual one but determined in such a way as to reproduce the relation between the node and the source existent in the initial real complex configuration network.

In the real network, the relation between the source and any node is established on the feed path of the latter. However, in the case of an arborescent configuration, this feed path is formed of a numerous branches, the majority of them also being part of other nodes' feed paths. The problem arises when separating for each branch of the real network, the effect produced by the load of each of supplied nodes through that branch. In other words, to find the fictitious branches (through their equivalent parameters) on the feed path for every load, on condition that the effects of their being traversed by the individual load to be identical with those traversing the real feed path, in the presence of the other loads. Finally, the fictitious lines linked individually to the source of the network's nodes resulted from the transfiguration, are to be obtained by summing up the individual fictitious branches existing in that feed path.

The transfiguration problem is therefore reduced to a series of secondary problems, each secondary problem consisting of establishing the fictitious branch from the feeding path associated to the real (or equivalent) load of the node in question. In order to solve each secondary problem, one must identify in the real arborescent network (fig.1a), the feed path in question and the entire network being reduced to this one. This operation will include the reduction of the ramifications to the feed path to the real and reactive powers at the beginning of the ramifications. In figure 1.b, it is presented as an example, the feed path of the node *n*.

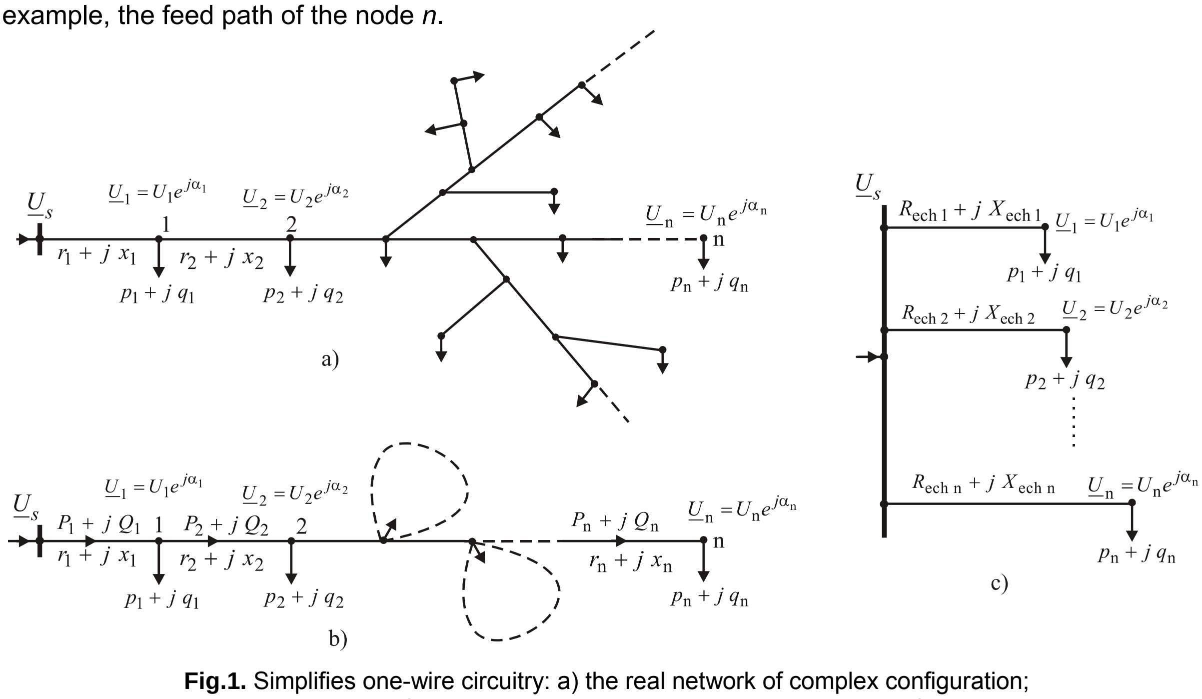

Fig.1. Simplifies one-wire circuitry: a) the real network of complex configuration; b) the real network reduced to the feed path; c) the network obtained through transfiguration, with individual feeders.

When solving the problem, there are to be used the ordinary approximations when dialing with the analysis of distribution networks, i.e. neglecting the transversal equivalent parameters of the electrical lines (the conductance and the susceptance) and approximating the real and the reactive powers respectively, flowing on a branch, with the sum of the real and reactive loads respectively, fed through that branch.

3. The equivalent transfiguration from the point of view of voltage drops

The equivalent parameters of the fictitious individual lines from the network obtained after transfiguration are determined in such a way as the nodes should have the same voltages (as rms values and phase-shift angles) as in the real network. More precisely, in such a way as the voltage drops in both feeding paths, real and fictitious, should be equal.

In the real network, the total voltage drops produced on the feeding path of a given load *n* (fig. 1b) is written as the sum of the voltage drops on the branches which are part of the feeding path:

$$\Delta\underline{U}_n = \sum_{l=1}^{n} \Delta\underline{U}_l \approx \sum_{l=1}^{n} \Delta U_l + j\sum_{l=1}^{n} \delta U_l = \Delta U_n + j\cdot\delta U_n \tag{1}$$

where for the branch of index *l,* of equivalent parameters r_l and x_l, traversed by the real power P_l and the reactive power Q_l, the longitudinal components and transversal ones of the voltage drops are:

$$\Delta U_l = \frac{P_l\cdot r_l + Q_l\cdot x_l}{U_l}, \qquad \delta U_l = \frac{P_l\cdot x_l - Q_l\cdot r_l}{U_l} \tag{2}$$

On the fictitious individual line which feeds the load *n* (fig. 1c), having the equivalent parameters $R_{n\,ech}$ and $X_{n\,ech}$, is produced the voltage drop:

$$\Delta\underline{U}_{n\,\mathrm{ech}} = \frac{1}{U_n}\left[(p_n R_{n\,\mathrm{ech}} + q_n X_{n\,\mathrm{ech}}) + j(p_n X_{n\,\mathrm{ech}} - q_n R_{n\,\mathrm{ech}})\right] \tag{3}$$

The unknown parameters $R_{n\,ech}$ and $X_{n\,ech}$ are found by solving the system of two linear equations with two unknown terms, built through the equalization of the real parts and of the imaginary ones in the relations (3) an (1) respectively:

$$\begin{cases} \dfrac{p_n}{U_n}R_{n\,\mathrm{ech}} + \dfrac{q_n}{U_n}X_{n\,\mathrm{ech}} = \Delta U_n \\ \dfrac{p_n}{U_n}X_{n\,\mathrm{ech}} - \dfrac{q_n}{U_n}R_{n\,\mathrm{ech}} = \delta U_n \end{cases} \tag{4}$$

The solutions are:

$$R_{n\,\mathrm{ech}} = U_n\frac{p_n\cdot\Delta U_n - q_n\cdot\delta U_n}{p_n^2+q_n^2}, \quad X_{n\,\mathrm{ech}} = U_n\frac{p_n\cdot\delta U_n + q_n\cdot\Delta U_n}{p_n^2+q_n^2} \tag{5}$$

By introducing in the relation (5) the form (2) of the voltage drop components and then grouping by terms r_l respectively x_l, we obtain the interdependence relations between the equivalent parameters and the real parameters of the branches from the real network:

$$\begin{aligned} R_{n\,\mathrm{ech}} &= \sum_{l=1}^{n}\frac{U_n}{U_l}\frac{(p_nP_l+q_nQ_l)}{s_n^2}\cdot r_l + \sum_{l=1}^{n}\frac{U_n}{U_l}\frac{(p_nQ_l-q_nP_l)}{s_n^2}\cdot x_l \\ X_{n\,\mathrm{ech}} &= \sum_{l=1}^{n}\frac{U_n}{U_l}\frac{(p_nP_l+q_nQ_l)}{s_n^2}\cdot x_l + \sum_{l=1}^{n}\frac{U_n}{U_l}\frac{(q_nP_l-p_nQ_l)}{s_n^2}\cdot r_l \end{aligned} \tag{6}$$

It can be noticed that both the equivalent resistance and the equivalent reactance of the fictitious line depend both on the resistances as well as on the reactances of the real branches and of course on the real and reactive powers flowing through these. We can write this interdependence by means of some weighting factor:

$$R_{n\,\mathrm{ech}} = \sum_{l=1}^{n} C_{ln}\cdot r_l + \sum_{l=1}^{n} C_{R\,ln}\cdot x_l, \qquad X_{n\,\mathrm{ech}} = \sum_{l=1}^{n} C_{ln}\cdot x_l + \sum_{l=1}^{n} C_{X\,ln}\cdot r_l \tag{7}$$

Their analytical expressions are obvious and their physical significances are as follows:

C_{nl} - the weighting factor of the resistance and reactance of the branch *l*, when calculating the resistance and the reactance of the fictitious individual line corresponding to the node *n*; if the variations of the voltage in the network's nodes are neglected ($U_n \approx U_l$), we can write:

$$C_{ln} = \frac{p_nP_l + q_nQ_l}{s_n^2} \tag{8}$$

$C_{R\,ln}, C_{X\,ln}$ - the weighting factor of the reactance and resistance of the branch's *l* respectively, when calculating the resistance and the reactance respectively of the fictitious individual line corresponding to the node *n*.

It is noticed that $C_{R\,ln} = -C_{X\,ln}$. Moreover, in the particular case when the power factors of all the loads are equal, the same value will be found for the power factors of the real branches:

$$tg\varphi_n = \frac{q_n}{p_n} = \frac{Q_l}{P_l} = tg\varphi_n \quad (9)$$

so that:

$$q_n P_l - p_n Q_l = q_n P_l \left(1 - \frac{tg\varphi_l}{tg\varphi_n} \right) = 0 \quad (10)$$

and therefore:

$$C_{R\,l\,n} = -C_{X\,l\,n} = 0 \quad (11)$$

In this particular case, the equivalent resistances and reactances respectively of the individual line from the network resulted after the transfiguration depends only on the resistances and on the reactances respectively of the branches of the feeding path in the real network.

It is also noticed that the weighting factors C_{ln} of the resistance and the reactance of a branch are always exceed unity, increasing in value while the load *n* decreases beside to the sum of the other loads fed through the branch *l*. For the branches that feed a single load, they become unitary, of course.

4. The equivalent transfiguration from the point of view of the real power losses

The equivalent parameters of a fictitious individual line in the network resulted from the transfiguration are determined in such a way as the real power losses produced by the individual load fed through this one, be equal with that produced by the same load on the feeding path in the real network, in the presence of the other loads.

In order to establish the mathematical model, we consider again the feeding path of the load *n* from the real network (fig. 1b). For its first element, of r_1 resistance, we write the equality between the losses produced by the real total load $\underline{S}_1 = P_1 + jQ_1$ on this element and the sum of the losses produced by the loads fed through element 1, noted $\underline{s}_k$ $(k = \overline{1,n}$, which traverse individually the fictitious elements, of resistances ρ_{1k}, $(k = \overline{1,n})$:

$$\frac{P_1^2 + Q_1^2}{U_1^2} r_1 = \frac{p_1^2 + q_1^2}{U_1^2} \rho_{11} + \frac{p_2^2 + q_2^2}{U_1^2} \rho_{12} + \ldots + \frac{p_n^2 + q_n^2}{U_1^2} \rho_{1n} \quad (12)$$

or:

$$s_1^2 \cdot \rho_{11} + s_2^2 \cdot \rho_{12} + \ldots + s_n^2 \cdot \rho_{1n} = S_1^2 \cdot r_1 \quad (13)$$

The fictitious resistances ρ_{1k} $(k = \overline{1,n})$ corresponding to the first real branch are determined as being dependent on its real resistance, by mean of some weighting factors:

$$\rho_{1k} = c_{1k} \cdot r_1, \quad k = \overline{1,n} \quad (14)$$

Thus, the equivalent resistance of the individual line that feeds the load *n* in the network obtained after transfiguration, will be the sum of the fictitious components ρ_{1k} $(k = \overline{1,n})$ corresponding to the *l* element on the real feeding path:

$$R_{n\,\mathrm{ech}} = \sum_{l=1}^{n} \rho_{ln} = \sum_{l=1}^{n} c_{ln} \cdot r_l \quad (15)$$

In order to determine the fictitious resistances ρ_{1k} $(k = \overline{1,n})$ we develop the left part of the relation (13) and we group the members. We obtain:

$$\left(p_1^2 \frac{p_1 + p_2 + \ldots + p_n}{p_1} + q_1^2 \frac{q_1 + q_2 + \ldots + q_n}{q_1} + p_2^2 \frac{p_1 + p_2 + \ldots + p_n}{p_2} + q_2^2 \frac{q_1 + q_2 + \ldots + q_n}{q_2} + .. \right.$$
$$\left. .. + p_n^2 \frac{p_1 + p_2 + \ldots + p_n}{p_n} + q_n^2 \frac{q_1 + q_2 + \ldots + q_n}{q_n} \right) \cdot r_1 = S_1^2 \cdot r_1 \quad (16)$$

or:

$$s_1^2 \frac{p_1 P_1 + q_1 Q_1}{s_1^2} \cdot r_1 + s_2^2 \frac{p_2 P_1 + q_2 Q_1}{s_2^2} \cdot r_1 + \ldots + s_k^2 \frac{p_k P_1 + q_k Q_1}{s_k^2} \cdot r_1 + \ldots + s_n^2 \frac{p_n P_1 + q_n Q_1}{s_n^2} \cdot r_1 = S_1^2 \cdot r_1 \quad (17)$$

Comparing now the relations (13) and (17) we find the expression of the fictitious resistance of network element 1, traversed by the individual charge *k* and of the corresponding weighting factor:

$$\rho_{1k} = \frac{p_k P_1 + q_k Q_1}{s_k^2} \cdot r_1, \qquad c_{1k} = \frac{p_k P_1 + q_k Q_1}{s_k^2} \tag{18}$$

and for the general case of branch *l* on the feeding path of the load *n*, results:

$$\rho_{l\,n} = \frac{p_n P_l + q_n Q_l}{s_n^2} \cdot r_l, \qquad c_{l\,n} = \frac{p_n P_l + q_n Q_l}{s_n^2} \tag{19}$$

It can be noticed that the value of the equivalent parameters of the individual lines of the network resulted after the equivalent transfiguration from the point of view of the real power losses, have very close values to these of the parameters resulted from the equivalent transfiguration from the point of view of voltage drops. For the particular case of the same power factor to all the loads, results:

$$c_{l\,n} = C_{l\,n} \tag{20}$$

which means that in this particular case, the equivalent transfiguration from the point of view of the voltage drops is valid also from the point of view of the active power losses.

The identification of the equivalent resistances of the individual lines leads in fact to the identification of the every load contribution to the total real power losses in the network, an extremely useful tool in many practical applications.

The equivalent reactances in the network obtained after the transfiguration are to be calculated similarly to the equivalent resistances, the weighting factors being identical. Thus, it is also obtained equivalence from the point of view of reactive power losses. This statement is valid only if the transversal equivalent parameters of the elements in the real network are neglected. If this hypothesis cannot be applied, in order for the transfiguration to be free of errors, the transversal equivalent parameters must be transformed previously in fictitious loads, connected at the ends of the longitudinal equivalent elements.

5. Application

To validate the methods above, a 20 kV nominal voltage, arborescent tree-phase network is considerate, build with single phase polyethylene insolated cables.

In figure 2 is presented the single-wire schema of the network, containing 25 buses (13 consumption buses) and 24 branches. The values of the equivalent longitudinal parameters of the real network's branches are presented in table 1, and the values of the active and reactive power of the consumptions buses are shown in figure 2. On the same figure there are written the rms values and phases for the voltages in the consumption buses and also the power losses on the branches. These values are resulted using a dedicated program for the calculus of the power flow in normal operating conditions [7].

Table 1. Longitudinal equivalent parameters of the real network branches.

Branch	*R* [Ω]	*X* [Ω]	Branch	*R* [Ω]	*X* [Ω]	Branch	*R* [Ω]	*X* [Ω]
NE-2	0,1940	0,1090	7-9	0,2448	0,0936	14-c_6	0,1528	0,0585
2-3	0,1836	0,0702	9-c_5	0,1528	0,0585	17-c_8	0,1528	0,0585
2-6	0,1940	0,1090	9-c_4	0,2140	0,0819	17-c_9	0,2448	0,0936
3-c_2	0,2448	0,0936	12-13	0,1446	0,0702	c_10-20	0,1224	0,0468
c_1-3	0,2740	0,1053	12-20	0,1684	0,0784	20-22	0,1836	0,0702
6-7	0,2448	0,0936	13-14	0,1836	0,0702	20-c_11	0,2752	0,1053
6-12	0,1204	0,0560	13-17	0,0916	0,0348	22-c_13	0,1528	0,0585
c_3-7	0,1224	0,0468	14-c_7	0,2448	0,0936	22-c_12	0,2140	0,0819

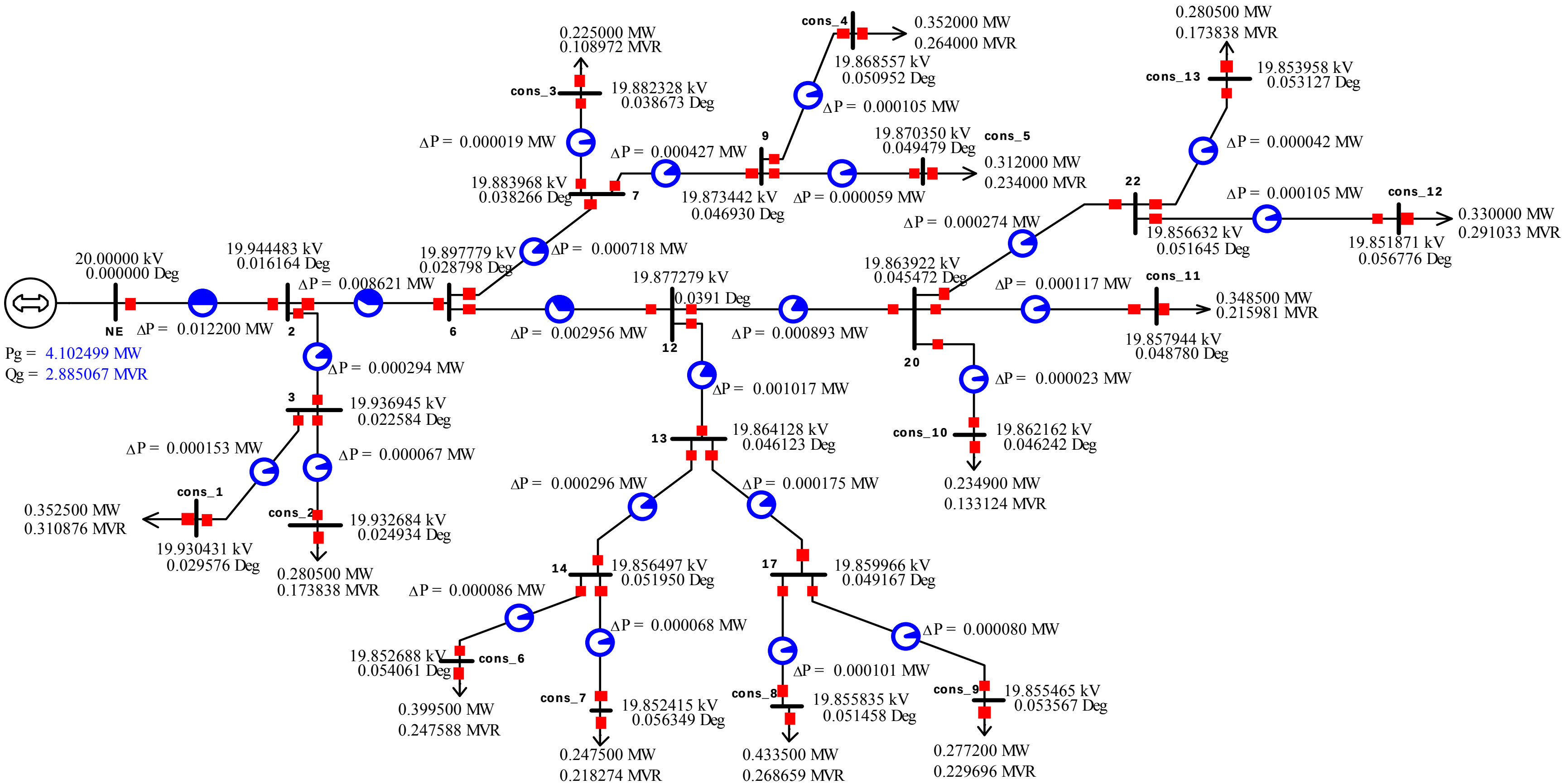

Fig. 2 The single-wire schema of the network studied, and the normal operating conditions.

a. The equivalence from the point of view of the voltage drops.

For verification of the correctness of the real network transfiguration in a fictitious network, equivalent from the point of view of voltage drops, the equivalent parameters computed using (6) – table 2 – were introduced in the network from figure 3, on which was applied the calculus of the power flow.

Table 2.
The resistances and reactances of the lines for the equivalance netwoek fom the point of view of voltage drops.

Equivalent line	R_{ech} [Ω]	X_{ech} [Ω]
1	2,501300	1,623990
2	3,732330	1,696410
3	8,888690	3,117250
4	5,227920	2,921880
5	5,810060	3,262820
6	5,700750	2,470910
7	7,450150	4,793070
8	5,135010	2,277030
9	6,798010	4,163230
10	9,404480	3,936340
11	6,280680	2,847930
12	5,607480	3,600500
13	8,041550	3,519310

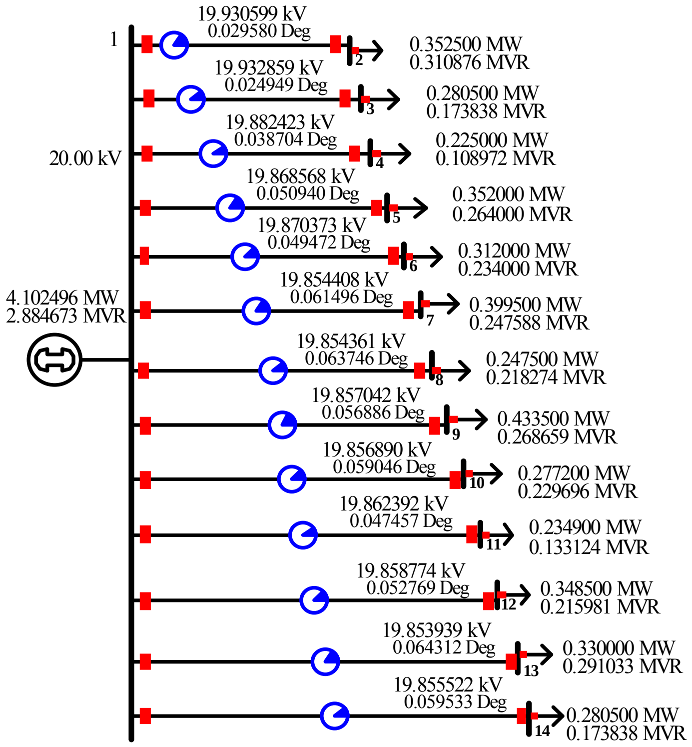

Fig. 3. The equivalence network from the point of view of voltage drops

Analyzing the network, we remark a good equivalence from the point of view of rms values of the voltages in the consumption buses, the maximal error is about 0,1%. For phases values the errors are greater (maxim 13,2%), due to the very low absolute values of the phases.

b. The equivalence from the point of view of the active power losses.

In table 3 are presented the parameters of the lines from the fictive network, equivalent from the point of view of active power losses produced on the supply way, computed using (19). With these values, was made the equivalent network. Also in table 3 are presented the active power losses on each equivalent line from the calculus of the active power flow in the equivalent network. On this way we can find the participation of all consumptions at these losses, which is useful in many practical applications.

In table 4 are presented the total active losses in the real network and in the equivalent network from the point of view of the active power losses, respective the equivalence error. We observe that the equivalence errors are very small, lower that 0,5%, which confirm the correctness of the equivalence.

6. Conclusions

The present paper deals with finding the mathematical model that allow overcome the difficulties due to the complex configurations of electrical distribution network and thus considerable simplification of the effort involved when doing the necessary analysis when applying the optimization method of the network operating state. More precisely, an original concept is defined, where a fictitious network with a simple radial configuration where each load node is fed through an individual line, can replace a real distribution network, having an arborescent configuration no matter how complex. The equivalence thus obtained is valid either from the point of view of voltage drops or from that of real power losses, a particular case when the equivalence is valid according to both criteria also existing. Therefore, this method allows for every load bus of a distribution network to be associated a fictitious line linked directly to the source, line of which equivalent parameters are established in dependence of the loads and the topology of the real network. Thus, the difficulty introduced in the optimization problems by the complex arborescent configuration is overcome, as this is integrated in the values of the parameters of the fictitious equivalent lines.

Table 3. The resistances of the lines of the equivalence network from the point of view of power losses and the losses produced on them.

Equivalent line	R_{ech} [Ω]	ΔP [kW]	Equivalent line	R_{ech} [Ω]	ΔP [kW]
1	2,6440	1,470	8	5,0180	3,311
2	3,6300	0,995	9	7,1243	2,341
3	8,2311	1,301	10	9,0032	1,664
4	5,3381	2,618	11	6,1405	2,618
5	5,9339	2,286	12	5,9709	2,932
6	5,5736	3,124	13	7,8652	2,173
7	7,9288	2,190			

Table 4. Total active power losses in real network and in the equivalence network from the point of view of real power losses, respective the equivalence error.

	ΔP [kW]	$\varepsilon_{\Delta P}$ [%]
Real network	28,89857	-
Equivalent network	29,02150	0,425

At the verification on a real network, the methods have conduced to very small equivalence errors, which confirm the correctness of the mathematical model.

We can observe that the proposed equivalence method can be applied also for only an area of the network or for only one or many buses, not perforce consumption buses.

If we consider the variation of the load, the equivalent parameters will have the same variation, which, as we can observe, is easy to establish and thus the equivalence methods proposed here, permits a simplification of optimization problems for the operating conditions.

The next step of this paper is to apply the equivalence methods presented here for the solutions of the problems regarding the voltage control, the placement of the reactive power sources, flattening of load curves, reconfiguration etc, and to compare the results and the efficacy of the calculus with that obtained by classical methods.

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