VOLTAGE DROP AND POWER CAPACITY COMPUTATION OF AN UNBALANCED DISTRIBUTION OVERHEAD LINE USING ACCURATE THREE-PHASE MODEL

G. Živadinović, PD Elektrodistribucija-Beograd, d.o.o, Serbia & Montenegro

1. INTRODUCTION

Most power systems textbooks and courses are limited to the modeling and analysis of balanced three-phase systems. The models and analyses assume a balance so that only a single-phase equivalent model is required. While this model gives satisfactory results for large interconnected system, it is not sufficient for the modeling and analysis of a distribution system. A distribution system is inherently unbalanced, and therefore three-phase models of all the components must be employed.

This paper gives an example of computation of voltage drop of each phase and maximum power that can be transmmited by an untransposed distribution overhead line using accurate three-phase model, based on the phase impedance and admittance matrices.

High-voltage transmission lines are usually transposed. Because distribution systems consist of three-phase lines serving unbalanced loads, it is necessary to retain the identity of the equivalent single-phase impedance terms of the conductors and to take into account the ground return path using Carson's equations.

An example of computation of the overhead line is given in the paper. The 35 kV distribution line, constructed with 18m concrete-reinforced poles, and with delta-configured three-phase 70/12 mm^2 conductors is considered. The results of calculation are three different voltage drops of each phase. The coefficient of unbalance is introduced to show the measure of symmetry of voltages at sending end of the line, assuming that the three-phase voltages at source end are equal and symmetric. This coefficient is defined according to American National Electrical Manufacturers Association standard as quotient of maximum deviation from average value of three-phase voltages to average value. The power transmission capacity of an overhead line depends on length of a line. Numerical example shows two different values of power transmission capacities of transposed and untransposed distribution line. Transmission capacity computated assuming transposed three-phase line is biger than that of untransposed line, which takes into account the maximum percent voltage drop of critical phase.

2. SERIES IMPEDANCE OF OVERHEAD LINES

The intent of this paper is to calculate voltage drop and power transmission capacity of the line using accurate three-phase model that consists of series impedances and shunt admittances. The determination of the series impedance for overhead and underground lines is a first step before the analysis of a distribution system can begin. The series impedance of a three-phase distribution line consists of the resistance of the conductors and self and mutual inductiv reactance resulting from the magnetic fields surrounding the conductors. Both resistances and reactances are usually expressed in Ω/km.

2.1 Transposed Three-Phase Lines

High-voltage transmission lines are usually assumed to be transposed, i. e. each phase occupies the same physical position on the structure for one-third of the length of the line. In addition to the assumption of transposition, it is assumed that the phases are equally loaded. With these two assumptions it is possible to combine the »self« and »mutual« terms into one »equivalent single-phase« impedance. The expression for phase impedance is:

$$\underline{z} = r + j \cdot 0.06283 \cdot \ln(\frac{D_{eq}}{GMR})\, \Omega/km, \quad D_{eq} = \sqrt[3]{D_{ab} \cdot D_{bc} \cdot D_{ca}} \tag{1.1}$$

where: D_{ab}, D_{bc} and D_{ca} are distances between phase conductors, $[m]$; D_{eq} - Geometric Mean Distance of conductors, $[m]$; r - conductor resistance, $[\Omega/km]$ and GMR - Geometric Mean Radius of phase conductor, $[m]$.

2.2 Untransposed Distribution Lines

Because distribution systems consist of untransposed three-phase lines serving unbalanced loads, it is necessary to retain the expression of the self and mutual impedance terms of the conductors and take into account the ground return path for the unbalance currents. Since distribution lines are inherently unbalanced, the most accurate analysis should not make any assumptions regarding the spacing between conductors, conductor sizes, and transposition. This can be done using Carson's set of equations that gives self and mutual impedances for an arbitrary number of overhead conductors. Carson's equations assume the earth is an infinite, uniform solid with a flat uniform upper surface and a constant resistivity. To apply Carson's equation, conductor images must be used. Every conductor at a given distance above ground has an image conductor the same distance below ground. This is illustrated in Figure 1.

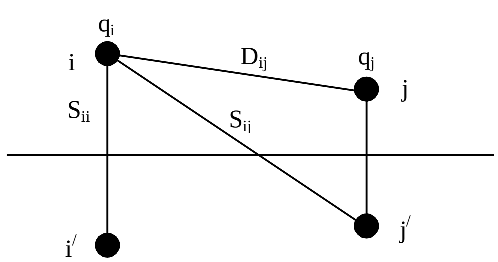

Figure 1 – Conductors and its images

If some approximations are made, modified Carson's equations result, which will be used in the paper. Those modified equations for self and mutual impedances are:

$$\underline{z}_{ii} = r_i + 0.049348 + j0.062832 \cdot \left(\ln \frac{1}{GMR_i} + 6.4905 + \frac{1}{2} \ln \frac{\rho}{f} \right) \Omega/km, \; i = (a,b,c) \tag{2.1}$$

for self impedances,

$$\underline{z}_{ij} = 0.049348 + j0.062832 \cdot \left(\ln \frac{1}{D_{ij}} + 6.4905 + \frac{1}{2} \ln \frac{\rho}{f} \right) \Omega/km, \; i,\, j = (a,b,c) \tag{2.2}$$

for mutual impedances.

In this equations f is frequency (50 Hz), and ρ is specific earth resistivity in $\Omega \cdot m$. Applying equations (2.1) and (2.2) a 3 x 3 phase impedance matrix for a three-phase line yields.

$$[\underline{z}_{abc}] = \begin{bmatrix} \underline{z}_{aa} & \underline{z}_{ab} & \underline{z}_{ac} \\ \underline{z}_{ba} & \underline{z}_{bb} & \underline{z}_{bc} \\ \underline{z}_{ca} & \underline{z}_{cb} & \underline{z}_{cc} \end{bmatrix} \ \Omega/km \tag{2.3}$$

In equation (2.3) diagonal terms $\underline{z}_{aa}$, $\underline{z}_{bb}$ and $\underline{z}_{cc}$ represent self-impedances of a line, while off-diagonal terms represent mutual-impedances. For a distribution line that is not transposed, the diagonal terms of equation (2.3) will not be equal to each other, and the off-diagonal terms will not be equal to each other. However, the matrix will be symmetrical.

Using phase impedance matrix, voltages at sending and receiving end of an overhead line can be expressed as:

$$\begin{bmatrix} \underline{V1}_{ag} \\ \underline{V1}_{bg} \\ \underline{V1}_{cg} \end{bmatrix} = \begin{bmatrix} \underline{V2}_{ag} \\ \underline{V2}_{bg} \\ \underline{V2}_{cg} \end{bmatrix} + \begin{bmatrix} \underline{Z}_{aa} & \underline{Z}_{ab} & \underline{Z}_{ac} \\ \underline{Z}_{ba} & \underline{Z}_{bb} & \underline{Z}_{bc} \\ \underline{Z}_{ca} & \underline{Z}_{cb} & \underline{Z}_{cc} \end{bmatrix} \cdot \begin{bmatrix} \underline{I}_a \\ \underline{I}_b \\ \underline{I}_c \end{bmatrix} \tag{2.4}$$

where: $\underline{V1}_{ag}$ is phase (a)–to ground voltage at receiving end (toward distribution substation), $\underline{V2}_{ag}$ is the phase (a)-to ground voltage at sending end (toward load) of a line, and $\underline{I}_a$, $\underline{I}_b$, $\underline{I}_c$ are line currents of an overhead line. Equation (2.4) can be written in condensed form as:

$$[\underline{V1}_{abc-g}] = [\underline{V2}_{abc-g}] + [\underline{Z}_{abc}] \cdot [\underline{I}_{abc}] \tag{2.5}$$

where elements of matrix $[\underline{Z}_{abc}]$ are $\underline{Z}_{ij} = \underline{z}_{ij} \cdot L_{line};\ i, j = (a,b,c)$, and L_{line} is length of a line.

3. SHUNT ADMITTANCE OF OVERHEAD LINES

The shunt admittance of a line consists of the conductance and the capacitive susceptance. The conductance is usually ignored because it is very small compared to the capacitive susceptance.

The voltage drop between conductor i and ground can be expressed as:

$$V_{ig} = P_{ii} \cdot q_i + P_{ij} \cdot q_j$$

where q_i and q_j are charge densities on conductors i and j respectively, and P_{ii}, P_{ij} self and mutual potential coefficients defined as:

$$P_{ii} = 17.9836 \cdot ln \frac{S_{ii}}{RD_i} \ [km/\mu F] \tag{3.1}$$

$$P_{ij} = 17.9836 \cdot ln \frac{S_{ij}}{D_{ij}} \ [km/\mu F] \tag{3.2}$$

In preceding equations (3.1) and (3.2) the terms have folowing meaning:

- S_{ii} = Distance from conductor i to its image i', $[m]$,
- S_{ij} = Distance from conductor i to the image of conductor j, designated in Figure 1 as j', $[m]$,
- D_{ij} = Distance from conductor i to conductor j, $[m]$,
- RD_i = Radius of conductor i, $[m]$.

For a three-phase overhead line, the 3 x 3 potential coefficient matrix can be constructed, using expressions (3.1) and (3.2). Thus, the following matrix is formed

$$[P_{abc}] = \begin{bmatrix} P_{aa} & P_{ab} & P_{ac} \\ P_{ba} & P_{bb} & P_{bc} \\ P_{ca} & P_{cb} & P_{cc} \end{bmatrix}$$

The inverse of the potential coefficient matrix will give 3 x 3 capacitance matrix $[C_{abc}] = [P_{abc}]^{-1}$.

Neglecting the shunt conductance, the phase shunt admittance matrix is given by:

$$\left[\underline{y}_{abc}\right] = 0 + j \cdot \omega \cdot \left[C_{abc}\right] \quad \mu S / km$$

Finally, total phase admittance matrix is defined as product of matrix $\left[\underline{y}_{abc}\right]$ and length of a line L_{line}:

$$\left[\underline{Y}_{abc}\right] = \left[\underline{y}_{abc}\right] \cdot L_{line}$$

4. EXACT DISTRIBUTION OVERHEAD LINE MODEL

The exact model of a three-phase overhead line is shown in Figure 2.

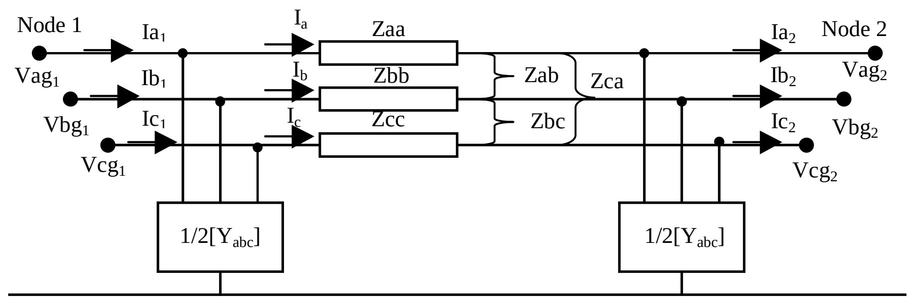

Figure 2 – Three-phase line segment model

Kirchhoff's current law in matrix form applied at node 2 of the model of Figure 2 gives:

$$\left[\underline{I}_{abc}\right] = \left[\underline{I}_{abc}\right]_2 + \frac{1}{2}\left[\underline{Y}_{abc}\right] \cdot \left[\underline{V}_{abc-g}\right]_2 \tag{4.1}$$

Kirchhoff's voltage law in matrix form applied to the model gives:

$$\left[\underline{V}_{abc-g}\right]_1 = \left[\underline{V}_{abc-g}\right]_2 + \left[\underline{Z}_{abc}\right] \cdot \left[\underline{I}_{abc}\right] \tag{4.2}$$

Substituting Equation (4.1) into Equation (4.2) and collecting terms yields:

$$\left[\underline{V}_{abc-g}\right]_1 = \left\{\left[U\right] + \frac{1}{2}\left[\underline{Z}_{abc}\right] \cdot \left[\underline{Y}_{abc}\right]\right\} \cdot \left[\underline{V}_{abc-g}\right]_2 + \left[\underline{Z}_{abc}\right] \cdot \left[\underline{I}_{abc}\right]_2 \tag{4.3}$$

where
$$\left[U\right] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Equation (4.3) is of the general form:

$$\left[\underline{V}_{abc-g}\right]_1 = \left[\underline{a}\right] \cdot \left[\underline{V}_{abc-g}\right]_2 + \left[\underline{b}\right] \cdot \left[\underline{I}_{abc}\right]_2 \tag{4.4}$$

where
$$\left[\underline{a}\right] = \left[\underline{U}\right] + \frac{1}{2}\left[\underline{Z}_{abc}\right] \cdot \left[\underline{Y}_{abc}\right]$$
$$\left[\underline{b}\right] = \left[\underline{Z}_{abc}\right]$$

The input current to the line at node 1 is

$$\left[\underline{I}_{abc}\right]_1 = \left[\underline{I}_{abc}\right] + \frac{1}{2}\left[\underline{Y}_{abc}\right] \cdot \left[\underline{V}_{abc-g}\right]_1 \tag{4.5}$$

Using Equations (4.1), (4.3) and (4.5) yields

$$\left[\underline{I}_{abc}\right]_1 = \left[\underline{c}\right] \cdot \left[\underline{V}_{abc-g}\right]_2 + \left[\underline{d}\right] \cdot \left[\underline{I}_{abc}\right]_2 \tag{4.6}$$

where $$[\underline{c}] = [\underline{Y}_{abc}] + \frac{1}{4}[\underline{Y}_{abc}]\cdot[\underline{Z}_{abc}]\cdot[\underline{Y}_{abc}]$$

$$[\underline{d}] = [U] + \frac{1}{2}[\underline{Z}_{abc}]\cdot[\underline{Y}_{abc}]$$

Equations (4.4) and (4.6) can be put into partitioned matrix form:

$$\begin{bmatrix} [\underline{V}_{abc-g}]_1 \\ [\underline{I}_{abc}]_1 \end{bmatrix} = \begin{bmatrix} [\underline{a}] & [\underline{b}] \\ [\underline{c}] & [\underline{d}] \end{bmatrix} \cdot \begin{bmatrix} [\underline{V}_{abc-g}]_2 \\ [\underline{I}_{abc}]_2 \end{bmatrix} \tag{4.7}$$

Equation (4.7) can be solved for the voltages and currents at node 2 in terms of the voltages and currents at node 1:

$$\begin{bmatrix} [\underline{V}_{abc-g}]_2 \\ [\underline{I}_{abc}]_2 \end{bmatrix} = \begin{bmatrix} [\underline{a}] & [\underline{b}] \\ [\underline{c}] & [\underline{d}] \end{bmatrix}^{-1} \cdot \begin{bmatrix} [\underline{V}_{abc-g}]_1 \\ [\underline{I}_{abc}]_1 \end{bmatrix} = \begin{bmatrix} [\underline{d}] & -[\underline{b}] \\ -[\underline{c}] & [\underline{a}] \end{bmatrix} \cdot \begin{bmatrix} [\underline{V}_{abc-g}]_1 \\ [\underline{I}_{abc}]_1 \end{bmatrix} \tag{4.8}$$

where identity $[\underline{a}]\cdot[\underline{d}]-[\underline{b}]\cdot[\underline{c}]=[U]$ is used. Since the matrix $[\underline{a}]$ is equal to the matrix $[\underline{d}]$, Equation (4.8) in expanded form becomes:

$$[\underline{V}_{abc-g}]_2 = [\underline{a}]\cdot[\underline{V}_{abc-g}]_1 - [\underline{b}]\cdot[\underline{I}_{abc}]_1 \tag{4.9}$$

$$[\underline{I}_{abc}]_2 = -[\underline{c}]\cdot[\underline{V}_{abc-g}]_1 + [\underline{d}]\cdot[\underline{I}_{abc}]_1 \tag{4.10}$$

Equations (4.9) and (4.10) will be used in numerical example of calculation in Section 5.

5. THE EXAMPLE OF VOLTAGE DROP COMPUTATION

This Section is devoted to give the numerical example of calculation of voltage drop using Equation (4.9) of the preceding Section. The 35kV overhead distribution line, designed with reinforced pole body and ACSR conductors in triangular (delta) configuration is considered.
For given length of an overhead line, values, such as three-phase line-to-line voltages, three-phase line-to-ground voltage drops, percent unbalance, and some other values is evaluated and shown as a function of input power, i.e. power at receiving end of the line.

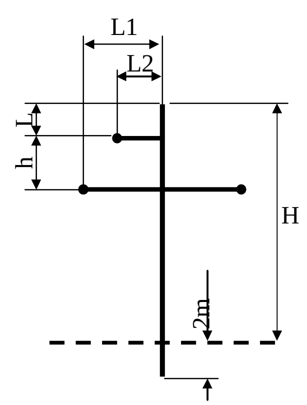

Figure 3 - An overhead line with conductors in triangular configuration

The balanced (symmetrical) three-phase voltages at receiving end of the line (toward to distribution substation X/35 kV) are assumed. Also the balanced customer demand is assumed. The distribution overhead line is not transposed, therefore the precise three-phase model of Equations (4.6) and (4.9) is used. Computation shows that the voltages at sending end of the line (towards customer) are not balanced.
The input data for the calculation are given in Table 1. Values L, $L1$, $L2$, h and H are referenced to Figure 3.

Table 2 simultaneously shows the results of computation of three-phase line-to-line voltages at sending end of the line using Equation (1.1) and Equation (4.9), for different values of input power. When calculation is performed using Equation (1.1), the balanced three-phase input voltages, balanced load, and transposed line are assumed. In this case a single-line equivalent circuit of a line is commonly used.

TABLE 1 - Input data for computation

No.	Input Data	Assumed Value
1.	Distance from top of the pole body to upper support (L)	0.2 m
2.	Length of upper support (L1)	1.6 m
3.	Length of lower support (L2)	1.25 m
4.	Heigth of the pole body (H)	18 m
5.	Difference of altitudes from upper to lower support (h)	2.8 m
6.	Type of conductors	ACSR 70/12 mm^2
7.	Geometric Mean Radius of conductor (GMR)	4.7478 mm
8.	Radius of conductor (RD)	5.85 mm
9.	Resistance of conductor per 1 km (r)	0.413 Ω/km
10.	Length of overhead line (L_{lline})	10 km
11.	Power Factor (cosφ)	0.95
12.	Rated line-to-line voltage of the line (V_{rated})	35 kV
13.	Specific resistivity of earth (ρ)	100 Ωm
14.	Frequency (f)	50 Hz
15.	Input line-to-line voltage (at receiving end of the line) ($V1_{rated}$)	36.5 kV
16.	Allowable percent voltage drop down the line (p%)	8%

Results of this evaluating are placed in column 2 of the table. Columns 3, 4, and 5 present the results of computation assuming untransposed distribution line giving the values of all phase voltages. Input power (kVA) that line receives from substation is placed in first column, varying from 0 to 10000 kVA. Using single-phase model of Equation (1.1) the impedance of the line of length 10 km is

$$\underline{Z} = (r + j \cdot 0.06283 \cdot \ln(\frac{D_{eq}}{GMR})) \cdot 10 = (4.130 + j \cdot 4.113)\ \Omega .$$

The three-phase accurate model, assuming untransposed line shown in Figure 3, gives the 3 x 3 phase impedance matrix:

$$\left[\underline{Z}_{abc}\right] = \begin{bmatrix} 4.623 + j7.657 & 0.493 + j3.644 & 0.493 + j3.565 \\ 0.493 + j3.644 & 4.623 + j7.657 & 0.493 + j3.426 \\ 0.493 + j3.565 & 0.493 + j3.426 & 4.623 + j7.657 \end{bmatrix} \Omega .$$

As can be seen the off-diagonal terms, which represent the mutual coupling between phases, are not equal to each other. In Table 2, values computated using Equation (1.1) are approximately the average values of those computating using Equation (4.9) for a given power. For example, at input power of 10000 kVA, the single-phase model voltage at sending end of the line is 35.081 kV, and this value is approximately equal to average value: $(35.065 + 35.127 + 35.043)/3 = 35.078\,kV$.

TABLE 2 - Results of computation using single-phase and accurate three-phase matrix models

Input Power (kVA)	Output Voltage Using Eq. (1.1) (kV)	Output Voltage of Phase “a” (kV)	Output Voltage of Phase “b” (kV)	Output Voltage of Phase “c” (kV)
0	36.500	36.498	36.498	36.498
1000	36.357	36.354	36.360	36.352
2000	36.215	36.210	36.223	36.206
3000	36.073	36.067	36.085	36.060
4000	35.930	35.923	35.948	35.914
5000	35.788	35.780	35.811	35.768
6000	35.647	35.637	35.674	35.623
7000	35.505	35.494	35.537	35.478
8000	35.363	35.351	35.400	35.333
9000	35.222	35.208	35.264	35.188
10000	35.081	35.065	35.127	35.043

Results of calculation of Table 2 (columns 3, 4 and 5), using three-phase accurate model, are shown in Figure 4. If voltage drops were calculated refering to rated line-to-ground voltage, the percent line-to-ground three-phase voltages would be evaluated, as shown in Figure 5.

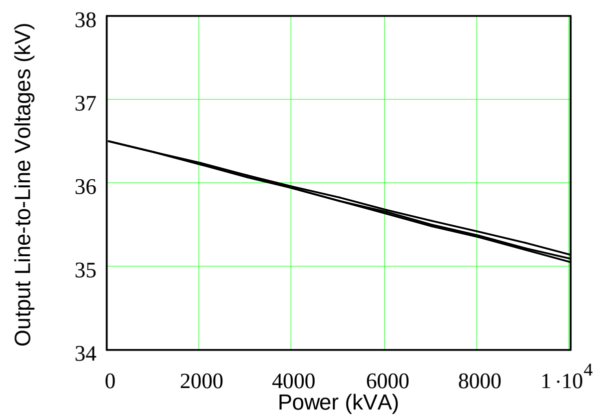

Figure 4 - Three-phase output (at sending end) line-to-line voltages in kV versus input power (at receiving end) in kVA.

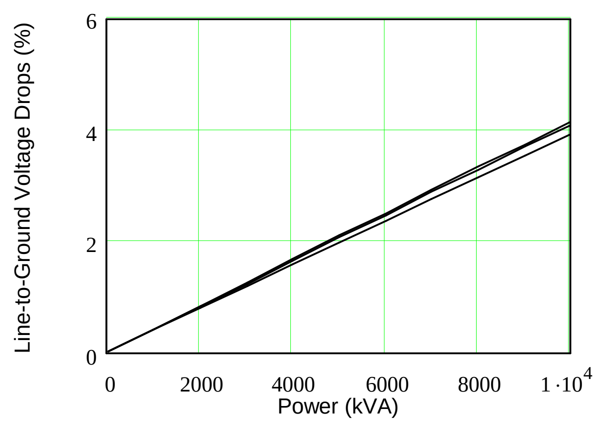

Figure 5 - Percent line-to-ground voltage drops referenced to rated line-to-ground voltage versus input power.

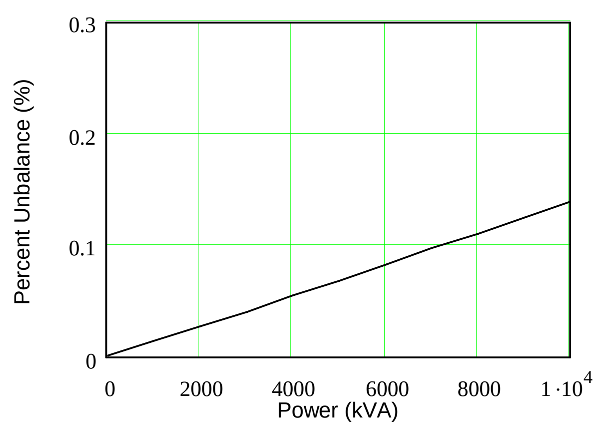

Figure 6 - Percent unbalance defined by Equation (5.1) as the function of the input power.

Because the mutual coupling between phases on the line are not equal, there will be different values of voltage drop on each of the three phases. As a result, the voltages on a distribution overhead line become unbalanced even when the loads and input set of voltages at the begining of the line are balanced.
A usefull method of describing the degree of unbalance is to use the following definition of unbalance factor:

$$Percent\ Unbalance\ Factor = \frac{|Maximum\ deviation\ \ from\ average|}{|V_{average}|} \cdot 100 \qquad (5.1)$$

Figure 6 illustrates the result of computation of Unbalance Factor as the function of input power at receiving end of the line for given input data of Table 1.
The residual of the section is devoted to calculation of Power Transmission Capacity of the overhead line, when the line is considered in, one case as transposed and with balanced loading, and in the other case as antransposed, with balanced load. In first case the Power Transmission Capacity, denoted as $S_{transposed}$, can be calculated using Equation (5.2)

$$S_{transposed}(MVA) = \frac{K}{L_{line}(km)}, \quad K = \frac{p\% \cdot V_{rated} \cdot V1_{rated}}{100 \cdot (\mathrm{Re}(z) \cdot \cos\varphi + \mathrm{Im}(z) \cdot \sqrt{1 - \cos\varphi^2})} \qquad (5.2)$$

where:
$p\%$ = maximum permissible percent voltage drop,
V_{rated} = rated line-to-line voltage,
$V1_{rated}$ = input line-to-line voltage,
$\underline{z}$ = impedance per 1 km of the balanced line, using single-phase equivalent model, Eq. (1.1),
$\cos\varphi$ = power factor of the load.

Figure 7 shows the difference between Power Transmission Capacity calculated by Equation (5.2), assuming transposed line (upper curve), and PTC calculated taking into consideration the maximum voltage drop of critical phase (lower curve). Horizontal straight line represents the maximum Power Capacity because of the limited value of line currents flowing through a given cross section of conductor used, (in this example 70/12 mm^2). Both curve give the maximum allowable power in MVA that can be transmited causing the voltage drop down the line less then or equal to 8%, versus length of the line in km.

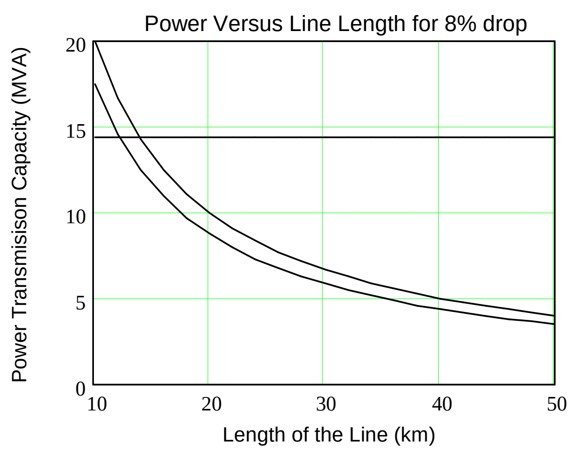

Figure 7. Power Transmission Capacity versus length of the line.
Upper curve represents the transposed line. Lower curve shows the untransposed distribution line.

6. CONCLUSION

Methods for computing series phase impedance and shunt capacitiv admittance matrices for overhead lines have been presented in this paper. Distribution lines are typically so short (compared to high-voltage transmission lines) that the shunt admittance can be ignored. However, there are cases of long, lightly loaded overhead lines where the shunt admittance should be included. Modified Carson's equations, that taking into account return path of currents through earth, are used in order to compute the phase impedances. When using the modified Carson's equations there is no need to make any assumptions, such as transposition of the lines. By assuming an untransposed line and including the actual phasing of the line, the most accurate values of the phase impedances and admittances are determined. Since voltage drop is a primary concern on a distribution line, the impedances used for the line must be as accurate as possible.

Three-phase output line-to-line voltages, thre-phase line-to-ground voltage drops, percent unbalance are calculated and shown in Figures 4, 5, and 6. Results shown in those Figures are obtained using exact model with no approximations; that is, assuming no transposition of the line.

Voltages at sending end of a distribution overhead line become unbalanced even when the loads and input set of voltages at the begining of the line are balanced.

The MATHCAD program for computation given in the paper is developed, making the possibility of voltage drop calculation varying all of the input data from Table 1.

Figure 7 shows the difference between Power Transmission Capacities of transposed and untransposed lines, with balanced input three-phase voltages and loading in both cases.

7. REFERENCES

[1] – William H. Kersting, “Distribution System Modeling and Analysis”, 2002, “CRC Press”, pages 77-141,

[2] – F. Kiessling, P. Nefzger, J. F. Nolasco, U. Kaintzyk, “Overhead Power Lines”, 2002, “Springer”,

[3] – J. D. Glover / M. Sarma, “Power System Analysis and Design”, (2nd ed.), 1994, “PWS-Kent Publishing”, Boston.